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IMPLEMENTING STEADY STATE EFFICIENCY IN OVERLAPPING GENERATIONS ECONOMIES WITH ENVIRONMENTAL EXTERNALITIES

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Abstract

We consider in this paper overlapping generations economies with pollution resulting from both consumption and production. The competitive equilibrium steady state is compared to the optimal steady state from the social planner's viewpoint. We show that the dynamical inefficiency of a competitive equilibrium steady state with capital–labor ratio exceeding the golden rule ratio still holds. Moreover, the range of dynamically efficient steady state capital ratios increases with the effectiveness of the environment maintenance technology, and decreases for more polluting production technologies. We characterize some tax and transfer policies that decentralize as a

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competitive equilibrium outcome the transition to the social planner's steady state.

1. Introduction

Environmental externalities have been studied in economies with overlapping generations for decades. In particular, the effects of environmental externalities on dynamic inefficiency, productivity, health, and longevity of agents have been addressed, as well as policy interventions that may be needed. While in most papers pollution is assumed to come from production, and the environment is supposed to improve or degrade by itself at a constant rate (Marini and Scaramozzino 1995; Jouvet, Michel, and Vidal 2000; Gutiérrez 2008; Pautrel 2009; Jouvet, Pestieau, and Ponthiere 2010), other papers assume that pollution comes from consumption (John and Pecchenino 1994; John *et al.* 1995; Ono 1996). As a consequence of the differing assumptions, accounts of the effect of environmental externalities on capital accumulation vary widely across papers. Specifically, John *et al.* (1995) showed that when only consumption pollutes, the economy accumulates less capital than what would be optimal. Conversely, Gutiérrez (2008) showed that when only production pollutes, the economy accumulates instead more capital than at the optimal level. This is so because in John *et al.* (1995) agents pay taxes to maintain environment when young, so that an increased pollution reduces their savings; however, in Gutiérrez (2008) pollution increases health costs in old age, leading agents to save more to pay for them. The difference seems therefore to come from when the taxes are paid (when young or old) rather than from whether pollution comes from production or consumption. Another main difference between John *et al.* (1995) and Gutiérrez (2008) is their different assumptions about the ability of environment to recover from pollution. John *et al.* (1995) assumes that environment naturally degrades over time, while Gutiérrez (2008) assumes that environment recovers naturally.

This paper aims at identifying the net impact of both production and consumption on environment by allowing for the two types of pollution simultaneously. Moreover, as in John and Pecchenino (1994) and John *et al.* (1995), we assume that the environment degrades naturally over time at a constant rate and that young agents devote part of their income to maintain it.[1] In this setup, we characterize the range of dynamically inefficient capital–labor ratios. Next, we introduce taxes and transfer policies that decentralize the first-best steady state, and the transition to it, as a competitive equilibrium steady state.

[1] In John and Pecchenino (1994) and John *et al.* (1995), only the consumption of old agents pollutes; young agents do not consume. In Ono (1996), it is assumed that consumption of both young and old agents degrade the environment but with a period lag. Here, we assume also that consumptions of both old and young agents and production pollute without decay.

The rest of the paper is organized as follows: Section 2 introduces the model. Section 3 characterizes its competitive equilibria. Section 4 presents the problem of the social planner, defines the efficient allocation with and without discounting, and characterizes the range of dynamically inefficient capital ratios (Proposition 1). The competitive equilibrium steady state and the planner's steady state are compared in Section 5 where we introduce some tax and transfer schemes that decentralize the planner's steady state and the transition to it as market outcome (from Proposition 2 to Proposition 5). Section 6 discusses these policies and show conditions under which a policy is both preferred by agents (Proposition 6), as well as more easily implementable. Section 7 concludes the paper.

2. The Model

We consider the overlapping generations economy in Diamond (1965) with a constant population of identical agents. At each period t output can be produced out of capital and labor according to a constant returns to scale neoclassical production function $F(K_t, L_t)$. This production is assumed to satisfy the condition $F_K(k,1) + F_{KK}(k,1)k > 0$ for all $k > 0$ where $k = K/L$, which guarantees the existence of competitive equilibrium dynamics (see Appendix A.3). Note that this property holds for all constant elasticity of substitution production functions $F(K_t, L_t) = [\theta K_t^\rho + (1-\theta)L_t^\rho]^{1/\rho}$, $\theta \in (0,1)$, $\rho \in (-\infty, 1]$. Capital fully depreciates in each period. The representative firm maximizes profits solving under perfect competition.

$$\max_{K_t, L_t \geq 0} F(K_t, L_t) - r_t K_t - w_t L_{t,}$$

so that the rental rate of capital and wage rate are, in each period t, the marginal productivity of capital and labor, respectively, i.e.,

$$r_t = F_K(K_t, L_t), \tag{1}$$

$$w_t = F_L(K_t, L_t). \tag{2}$$

The size of each generation is normalized to one. Each agent lives two periods, say young and old. When young, an agent is endowed with one unit of labor that he supplies inelastically, so that $L_t = 1$ for all t, since population is constant. Agents born in period t divide their wage w_t between consumption when young c_0^t, investment in maintaining the environment m^t, and savings k^t lent to firms for a return rate r_{t+1} to be used in $t+1$ as capital, so that $K_{t+1} = k^t$ since population is normalized to 1. The return to savings $r_{t+1}k^t$ is used up as old-age consumption. Agents born at date t have preferences over their consumptions when young and old $(c_0^t, c_1^t) \in \mathbb{R}_+^2$ and the environmental quality when old, $E_{t+1} \in \mathbb{R}$, represented by $u(c_0^t) + v(c_1^t) + \phi(E_{t+1})$ with u', v', $\phi' > 0$, u'', v'', $\phi'' < 0$, and $u'(0) = v'(0) = +\infty$, $u'(+\infty) = v'(+\infty) = 0$, $\phi'(+\infty) = 0$, $\phi'(-\infty) = +\infty$.[2]

Environmental quality evolves according to

$$E_{t+1} = (1-b)E_t - \alpha F(K_{t+1}, L_{t+1}) - \beta(c_0^{t+1} + c_1^t) + \gamma m^t \tag{3}$$

for some $\alpha, \beta, \gamma > 0$, and $b \in (0, 1]$. That is to say, environmental quality converges autonomously to a natural level normalized to zero at a rate b that measures the speed of reversion to this level. Nonetheless, production and consumption degrade environmental quality by an amount $\alpha F(K_{t+1}, 1)$ and $\beta(c_0^{t+1} + c_1^t)$, respectively, while young agents can improve the environmental quality they will enjoy when old by an amount γm^t if they devote a portion m^t of their labor income to that end.

The lifetime utility maximization problem of the representative agent is

$$\max_{c_0^t, c_1^t, k^t, m^t \geq 0 E_t,\, E_{t+1}^e} u(c_0^t) + v(c_1^t) + \phi(E_{t+1}^e) \tag{4}$$

subject to

$$c_0^t + k^t + m^t = w_t, \tag{5}$$

$$c_1^t = r_{t+1}k^t, \tag{6}$$

$$E_t = (1-b)E_{t-1} - \alpha F(k^{t-1}, 1) - \beta(c_0^t + c_1^{t-1}) + \gamma m^{t-1}, \tag{7}$$

$$E_{t+1}^e = (1-b)E_t - \alpha F(K_{t+1}, 1) - \beta(c_0^{t+1,e} + c_1^t) + \gamma m^t, \tag{8}$$

where E_{t+1}^e is, at t, the expected state of environment at $t+1$, given the expected consumption of the next generation young agent $c_0^{t+1,e}$ and E_{t-1}, c_1^{t-1}, k^{t-1}, m^{t-1}, w_t, r_{t+1}. Since the representative agent is assumed to be negligible within his own generation, he thinks the impact of his savings

[2] Green preferences are necessary to capture the externality, and this can be achieved in several ways. Many papers in the literature introduce the stock of pollution as a public bad entering the utility function instead of environmental quality. Alternatively, pollution makes agents pay a health cost that reduces utility because of consuming less. In our paper, following John and Pecchenino (1994) John *et al.* (1995), we introduce the environmental quality as a public good entering the utility function of agents. This approach to the impact of environment on choices of agents seems more adequate for several reasons. First, the health and utility of agents in fact depend on the quality of environment around rather than on the quantity of pollution. Second, from the dynamics of environmental quality in (3), environmental quality captures anyway the effect of pollution, and moreover not only the impact of current pollution and abatement activities but those in the past also. Third, without any activity the environmental quality converges (upgrades or depreciates itself) to a level normalized to zero; while for the alternative approach, environment just upgrades itself via a rate of decay of the stock of pollution, so that environment never degrades itself, which prevents capturing the reversion to wilderness that may render environment unfit for human activity.

k^t on aggregate capital K_{t+1} to be negligible as well, ignoring that actually $K_{t+1} = k^t$ at equilibrium. This assumption implies that he does not internalize the impact of the savings decision on environment via production. Notwithstanding, the agent considers the impact of his consumption and maintenance choices not to be negligible. This is meant to capture the idea that agents care actually, not for the global environment, but for the nearby environment on which their consumption and maintenance choices have a direct impact. In any event, given that there is a single representative agent, at equilibrium local and global environment coincide. Since production is not in the hands of the agent (although he supplies the necessary capital through his savings), that he disregards his impact on environment through production is the natural assumption to make.

An interior optimal choice $(c_0^t, c_1^t, k^t, m^t, E_t, E_{t+1}^e)$ for agent t is therefore characterized by the first-order conditions (FOCs)

$$u'(c_0^t) - [\beta(1-b) + \gamma]\,\phi'(E_{t+1}^e) = 0, \tag{9}$$

$$v'(c_1^t) - \left[\beta + \frac{\gamma}{r_{t+1}}\right]\phi'(E_{t+1}^e) = 0, \tag{10}$$

$$c_0^t + k^t + m^t - w_t = 0, \tag{11}$$

$$c_1^t - r_{t+1}k^t = 0, \tag{12}$$

$$E_t - (1-b)E_{t-1} + \alpha F(k^{t-1}, 1) + \beta(c_0^t + c_1^{t-1}) - \gamma m^{t-1} = 0, \tag{13}$$

$$E_{t+1}^e - (1-b)E_t + \alpha F(K_{t+1}, 1) + \beta(c_0^{t+1,e} + c_1^t) - \gamma m^t = 0, \tag{14}$$

to be an implicit function of E_{t-1}, c_1^{t-1}, k^{t-1}, m^{t-1}, w_t, r_{t+1}, and $c_0^{t+1,e}$ as long as the Jacobian matrix of the left-hand side of the system above with respect to c_0^t, c_1^t, k^t, m^t, E_t, E_{t+1}^e is regular at the solution. The existence and regularity of the optimal solution is established in Appendix A.1. For these FOCs to be not only necessary but also sufficient for the solution to be a maximum, the second-order conditions (SOCs) are shown to hold at equilibrium in Appendix A.2.

3. Competitive Equilibria

The perfect foresight competitive equilibria are characterized by (i) the agent's utility maximization under the budget constraints, with correct expectations, (ii) the firms' profit maximization determining factors' prices, and (iii) the dynamics of environment. Therefore, a competitive equilibrium allocation $\{c_0^t,\ c_1^t,\ k^t,\ m^t,\ E_{t+1}\}_t$ is a solution to the system of equations

$$u'(c_0^t) - [\beta(1-b) + \gamma]\,\phi'(E_{t+1}) = 0, \tag{15}$$

$$v'(c_1^t) - \left[\beta + \frac{\gamma}{F_K(k^t, 1)}\right]\phi'(E_{t+1}) = 0, \tag{16}$$

$$c_0^t + k^t + m^t - F_L(k^{t-1}, 1) = 0, \tag{17}$$

$$c_1^t - F_K(k^t, 1)k^t = 0, \tag{18}$$

$$E_{t+1} - (1-b)E_t + \alpha F(k^t, 1) + \beta(c_0^{t+1} + c_1^t) - \gamma m^t = 0. \tag{19}$$

Note that the feasibility of the allocation of resources is guaranteed by the agent's budget constraints (17) and (18), since at t

$$c_0^t + c_1^{t-1} + k^t + m^t = F_K(k^{t-1}, 1)k^{t-1} + F_L(k^{t-1}, 1) = F(k^{t-1}, 1).$$

The perfect foresight competitive equilibria of this economy follow a dynamics represented by a first-order difference equation, because of the regularity of the associated Jacobian matrix of the left-hand side of the system of equations above with respect to c_0^{t+1}, c_1^t, k^t, m^t, E_{t+1} (see Appendix A.3).

A perfect foresight competitive equilibrium steady state, in particular, is a (c_0, c_1, k, m, E) solution to the system of equations

$$u'(c_0) - [\beta(1-b) + \gamma]\phi'(E) = 0,$$

$$v'(c_1) - \left[\beta + \frac{\gamma}{F_K(k, 1)}\right]\phi'(E) = 0,$$

$$c_0 + k + m - F_L(k, 1) = 0,$$

$$c_1 - F_K(k, 1)k = 0,$$

$$bE + \alpha F(k, 1) + \beta(c_0 + c_1) - \gamma m = 0.$$

4. The Social Planner's Choice with and without Discounting

In this section, we consider the optimal allocation from the viewpoint of a social planner that allocates resources in order to maximize a weighted sum of the welfare of all current and future generations. The allocation selected by the social planner, which is optimal in the Pareto sense, is a solution to the problem

$$\max_{\{c_0^t, c_1^t, k^t, m^t, E_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \frac{1}{(1+R)^t}\left[u(c_0^t) + v(c_1^t) + \phi(E_{t+1})\right] \tag{20}$$

subject to, $\forall t = 0, 1, 2, \ldots,$

$$c_0^t + c_1^{t-1} + k^t + m^t = F(k^{t-1}, 1) \tag{21}$$

$$E_{t+1} = (1-b)E_t - \alpha F(k^t, 1) - \beta(c_0^{t+1} + c_1^t) + \gamma m^t, \tag{22}$$

given some initial conditions c_1^{-1}, k^{-1}, E_0, where $0 \leq R$ is the social planner's subjective discount rate.[3] The first constraint (21) of the problem is the resource constraint of the economy in period t requiring that the total output in that period is split into consumptions of the current young and old, savings for next period's capital, and environmental maintenance. The second constraint (22) is the dynamics of the environmental quality.

The social planner's choice of a steady state is a $(\bar{c}_0, \bar{c}_1, \bar{m}, \bar{k}, \bar{E})$ satisfying (see Appendix A.4)

$$u'(\bar{c}_0) = (1+R)\frac{\gamma + \beta(1+R)}{b+R}\phi'(\bar{E}), \tag{23}$$

$$v'(\bar{c}_1) = \frac{\gamma + \beta(1+R)}{b+R}\phi'(\bar{E}), \tag{24}$$

$$F_K(\bar{k}, 1) = \frac{\gamma(1+R)}{\gamma - (1+R)\alpha}, \tag{25}$$

$$\bar{c}_0 + \bar{c}_1 + \bar{k} + \bar{m} = F(\bar{k}, 1), \tag{26}$$

$$b\bar{E} + \alpha F(\bar{k}, 1) + \beta(\bar{c}_0 + \bar{c}_1) - \gamma\bar{m} = 0. \tag{27}$$

(the planner's discount rate R cannot be arbitrarily high for the optimal steady state to be characterized as above, specifically $\gamma > (1+R)\alpha$ needs to hold, which requires $\gamma > \alpha$, so that $F_K(\bar{k}, 1) > 0$). More specifically, in the case of the social planner caring about all generations equally, i.e., $R = 0$, the planer's steady state is the so-called golden rule steady state $\{c_0^*, c_1^*, k^*, m^*, E^*\}$ that maximizes the utility of the representative agent and is characterized by being a solution to the system

$$u'(c_0^*) = \frac{\gamma + \beta}{b}\phi'(E^*), \tag{28}$$

$$v'(c_1^*) = \frac{\gamma + \beta}{b}\phi'(E^*), \tag{29}$$

$$F_K(k^*, 1) = \frac{\gamma}{\gamma - \alpha}, \tag{30}$$

$$c_0^* + c_1^* + k^* + m^* = F(k^*, 1), \tag{31}$$

$$bE^* + \alpha F(k^*, 1) + \beta(c_0^* + c_1^*) - \gamma m^* = 0. \tag{32}$$

[3] The discount rate R is strictly positive when the social planner cares less about a generation's welfare the further away in the future that generation is, while R equals to zero when she cares about all generations equally, no matter how far in the future they may be.

Note that, from (28) and (29), the marginal utility of consumption of the young agent must equal that of the consumption of the old agent.

Diamond (1965) shows that, in the standard OLG model without pollution externalities, a competitive equilibrium steady state whose capital per worker exceeds the golden rule level is dynamically inefficient. In this paper, we consider instead an economy with pollution externalities coming from both production and consumption, in which the environment degrade itself over time, and where the quality of the environment can be improved through maintenance. It turns out that, as in Diamond (1965), the golden rule capital ratio of this economy with pollution externalities is still the highest level of capital ratio that is dynamically efficient.[4]

PROPOSITION 1: *In a Diamond (1965) overlapping generations economy with consumption and production pollution, for an efficient enough cleaning technology, compared to the marginal polluting impact of production (specifically, for $\gamma > \alpha$ in the model), the golden rule capital ratio (i.e., the planner's steady state choice without discounting) is the highest dynamically efficient capital ratio.*

Proof: Since $F_{KK}(k, 1) < 0$ for all k, the planner's optimal capital ratio $\bar{k}$ is implicitly defined to be a differentiable function $\bar{k}(R)$ of R by the condition

$$F_K(\bar{k}, 1) = \frac{\gamma(1+R)}{\gamma - (1+R)\alpha}$$

whose derivative, by the implicit function theorem, is

$$\bar{k}'(R) = \frac{1}{F_{KK}(\bar{k}(R), 1)} \left(\frac{\gamma}{\gamma - (1+R)\alpha}\right)^2 < 0. \tag{33}$$

So, $\bar{k}$ is decreasing in R. Hence, $\bar{k}(R)$ is maximal when $R = 0$, which is corresponds to the golden rule level of capital k^*. ■

Proposition 1 shows that any steady state capital ratio exceeding k^* is dynamically inefficient. From (30) the golden rule capital ratio k^* is decreasing in the production pollution parameter α. It is, however, increasing in the environment-maintaining technology γ. Hence, the more polluting is production, the smaller the range of steady state allocations that are dynamically efficient for some discount factor R. Similarly, the more effective is the

[4] In a different framework and when pollution externalities are large enough, Gutiérrez (2008) has shown the existence of dynamically efficient competitive equilibrium steady state capital ratios that exceed the golden rule capital ratio. Specifically, when (i) pollution externalities come only from production, (ii) environment recovers itself at a constant rate, (iii) no resource is devoted to maintaining the environment, and (iv) the pollution externality decreases the utility of the agents only indirectly by requiring each agent to pay for extra health costs in old age, Gutiérrez (2008) shows the existence of a "super golden rule" level of capital ratio (beyond the golden rule level) such that any stationary capital ratio exceeding this level is necessarily dynamically inefficient.

environment maintenance technology, the bigger the range of steady state allocations that are dynamically efficient for some discount factor R.

5. Policy Implementation of the Planner's Optimal Steady State

In this section, we provide tax and transfer policies allowing to implement the planner's optimal steady state. Ono (1996) and Gutiérrez (2008) also introduced tax and transfer schemes to decentralize the golden rule steady state in the context of the pollution externalities they consider (from consumption and production only, respectively). However, their schemes uphold the golden rule once the economy is already at that steady state. In this section, we provide instead policies that lead the economy toward the social planer's first-best steady state and will keep it there once reached. The policies fulfill this in two stages. In the first stage (in the period $t-1$), taxes and transfers are set in order to make the agent born in period $t-1$ choose $c_0^{t-1} = c_0^*$, $c_1^{t-1} = c_1^*$, $k^{t-1} = k^*$ and $E_t^e = E^*$ (but not $m^{t-1} = m^*$ or $E_{t-1} = E^*$). For the sake of avoiding unnecessarily cumbersome notation, the argument is presented for the case $R = 0$, although it can be rewritten for any $R \geq 0$). Then, in the second stage, taxes and transfers are reset to uphold the planner's steady state from period t onward. The first scheme based on the taxation of consumption is presented next in detail. The subsequent schemes work analogously.

5.1. Taxes on Consumptions

As in Ono (1996), we consider first taxes on consumption along with lump-sum taxes and transfers. Letting τ_0^t and τ_1^t be the tax rate on agent t's consumption when young and old, respectively, T_0^t a lump-sum tax (if positive) levied on agent t's income when young, and T_1^t a lump-sum transfer (if positive) to the same agent when old at date $t+1$, the problem of agent t is then

$$\max_{c_0^t, c_1^t, k^t, m^t \geq 0 E_t, E_{t+1}^e} u(c_0^t) + v(c_1^t) + \phi(E_{t+1}^e) \tag{34}$$

subject to

$$(1 + \tau_0^t) c_0^t + k^t + m^t = w_t - T_0^t, \tag{35}$$

$$(1 + \tau_1^t) c_1^t = r_{t+1} k^t + T_1^t, \tag{36}$$

$$E_t = (1 - b) E_{t-1} - \alpha F(k^{t-1}, 1) - \beta (c_0^t + c_1^{t-1}) + \gamma m^{t-1}, \tag{37}$$

$$E_{t+1}^e = (1 - b) E_t - \alpha F(K_{t+1}, 1) - \beta (c_0^{t+1,e} + c_1^t) + \gamma m^t, \tag{38}$$

given E_{t-1}, c_1^{t-1}, k^{t-1}, m^{t-1}, w_{t-1}, $c_0^{t+1,e}$, and r_{t+1}. Note again that in Equation (38), the agent, being negligible within his generation, ignores the fact that $K_{t+1} = k^t$ and hence is unable to internalize the effect of the savings decisions on environment through the aggregate output.

PROPOSITION 2: *In a Diamond (1965) overlapping generations economy with pollution from both consumption and production, the planner's steady state can be implemented at any given period t by the following period-by-period balanced-budget policy: Announce at $t-1$ that the following consumption tax rates and lump-sum transfers will apply,*

(1) to generation born at $t-1$

$$\tau_0^{t-1} = \frac{\gamma + \beta - [\gamma + \beta(1-b)]\, b}{\gamma b},$$

$$\tau_1^{t-1} = \frac{\gamma + \beta(1-b)}{(\gamma - \alpha) b} - 1,$$

$$T_0^{t-1} = \frac{1}{A}\begin{pmatrix} \gamma \\ 1-b \\ -1 \end{pmatrix}'$$

$$\begin{pmatrix} F_L(k^{t-2}, 1) - (1+\tau_0) c_0^* - k^* \\ (1-b) E_{t-2} - \alpha F(k^{t-2}, 1) - \beta((1+\tau_0) c_0^* + c_1^{t-2}) + \gamma m^{t-2} \\ E^* + \alpha F(k^*, 1) + \beta(c_0^* + c_1^*) \end{pmatrix},$$

$$T_1^{t-1} = (1+\tau_1) c_1^* - F_K(k^*, 1) k^*,$$

where $A = \gamma + \beta(1-b)$

(2) to generation born from t onwards

$$\tau_0 = \frac{\gamma + \beta - [\gamma + \beta(1-b)]\, b}{\gamma b},$$

$$\tau_1 = \frac{\gamma + \beta(1-b)}{(\gamma - \alpha) b} - 1,$$

$$T_0 = F_L(k^*, 1) - (1+\tau_0) c_0^* - k^* - m^*,$$

$$T_1 = (1+\tau_1) c_1^* - F_K(k^*, 1) k^*.$$

Proof: See Appendix A.5.

5.2. Taxes on Consumptions and Capital Income

In Section 5.1, we introduced taxes on consumptions in which the tax rates differ between consumptions of the old and the young. In reality, however, this tax scheme seems to be difficult to apply because it discriminates between young and old agents. In order to avoid the discrimination, a unique rate of consumption tax τ^t should be applied. Beside that, a capital income tax τ_k^t and a system of lump-sum tax T_0^t (if positive) and lump-sum transfer T_1^t (if positive), levied on agent t's incomes, are introduced to show that the best steady state allocation can be achieved. The problem of agent t under the tax policy is then

$$\max_{c_0^t, c_1^t, k^t, m^t \geq 0 E_t, E_{t+1}^e} u(c_0^t) + v(c_1^t) + \phi(E_{t+1}^e) \tag{39}$$

subject to

$$(1+\tau^t)c_0^t + k^t + m^t = w_t - T_0^t, \tag{40}$$

$$(1+\tau^t)c_1^t = (1-\tau_k^t)r_{t+1}k^t + T_1^t, \tag{41}$$

$$E_t = (1-b)E_{t-1} - \alpha F(k^{t-1}, 1) - \beta(c_0^t + c_1^{t-1}) + \gamma m^{t-1}, \tag{42}$$

$$E_{t+1}^e = (1-b)E_t - \alpha F(K_{t+1}, 1) - \beta(c_0^{t+1,e} + c_1^t) + \gamma m^t, \tag{43}$$

given E_{t-1}, c_1^{t-1}, k^{t-1}, m^{t-1}, w_t, $c_0^{t+1,e}$, and r_{t+1}. Note again that in (43), the agent, being negligible within his generation, ignores the fact that $K_{t+1} = k^t$ and hence is unable to internalize the effect of the savings decisions on environment through the aggregate output.

PROPOSITION 3: *In a Diamond (1965) overlapping generations economy with pollution from both consumption and production, the planner's steady state can be implemented at any given period t by the following period-by-period balanced-budget policy: Announce at $t-1$ that the following consumption tax rate, capital income tax rate and lump-sum transfers will apply,*

(1) to generation born at $t-1$

$$\tau^{t-1} = \frac{\gamma + \beta - [\gamma + \beta(1-b)]\, b}{\gamma b},$$

$$\tau_k^{t-1} = \frac{\alpha(\gamma+\beta) - \beta b^2(\gamma - \alpha) - \alpha\beta b}{\gamma\,[\gamma + \beta(1-b)]},$$

$$T_0^{t-1} = \frac{1}{A}\begin{pmatrix} \gamma \\ 1-b \\ -1 \end{pmatrix}'$$

$$\begin{pmatrix} F_L(k^{t-2}, 1) - (1+\tau)c_0^* - k^* \\ (1-b)E_{t-2} - \alpha F(k^{t-2}, 1) - \beta((1+\tau)c_0^* + c_1^{t-2}) + \gamma m^{t-2} \\ E^* + \alpha F(k^*, 1) + \beta(c_0^* + c_1^*) \end{pmatrix},$$

$$T_1^{t-1} = (1+\tau)c_1^* - (1-\tau_k)F_K(k^*, 1)k^*,$$

where $A = \gamma + \beta(1 - b)$,

(2) to all generations born from period t onwards

$$\tau = \frac{\gamma + \beta - [\gamma + \beta(1 - b)]\, b}{\gamma b},$$

$$\tau_k = \frac{\alpha(\gamma + \beta) - \beta b^2(\gamma - \alpha) - \alpha\beta b}{\gamma\,[\gamma + \beta(1 - b)]},$$

$$T_0 = F_L(k^*, 1) - (1 + \tau)c_0^* - k^* - m^*,$$

$$T_1 = (1 + \tau)c_1^* - (1 - \tau_k)F_K(k^*, 1)k^*.$$

Proof: The proof for this proposition is similar to the proof for Proposition 2. ■

5.3. Taxes on Consumptions and Production

We still keep the nondiscriminatory tax rate τ^t on consumptions and the system of lump-sum tax T_0^t (if positive) and lump-sum transfer T_1^t (if positive). However, we now introduce a Pigouvian tax on production instead of tax on capital income. In any period t, let τ_p^t be the tax paid by firms per one unit of output produced in period t. We will design taxes and transfers policy ensuring the government's budget to be balanced and achieving the planner's steady state through competitive markets.

Under the production tax, the problem that the firm must solve in period t is

$$\max_{K_t} \quad (1 - \tau_p^t)F(K_t, 1) - r_t K_t - w_t. \tag{44}$$

The returns to capital and labor are at equilibrium, respectively,

$$r_t = (1 - \tau_p^t)F_K(k^{t-1}, 1), \tag{45}$$

$$w_t = (1 - \tau_p^t)F_L(k^{t-1}, 1). \tag{46}$$

Under the taxes and transfers policy, the agent t's problem is

$$\max_{c_0^t, c_1^t, k^t, m^t \geq 0 E_t,\, E_{t+1}^e} u(c_0^t) + v(c_1^t) + \phi(E_{t+1}^e) \tag{47}$$

subject to

$$(1 + \tau^t)c_0^t + k^t + m^t = w_t - T_0^t, \tag{48}$$

$$(1 + \tau^t)c_1^t = r_{t+1}k^t + T_1^t, \tag{49}$$

$$E_t = (1 - b)E_{t-1} - \alpha F(k^{t-1}, 1) - \beta(c_0^t + c_1^{t-1}) + \gamma m^{t-1}, \tag{50}$$

$$E^e_{t+1} = (1-b)E_t - \alpha F(K_{t+1},1) - \beta(c_0^{t+1,e} + c_1^t) + \gamma m^{t-1}, \tag{51}$$

given E_{t-1}, c_1^{t-1}, k^{t-1}, m^{t-1}, w_t, $c_0^{t+1,e}$, and r_{t+1}. Note again that in (51), the agent, being negligible within his generation, ignores the fact that $K_{t+1} = k^t$ and hence is unable to internalize the effect of the savings decisions on environment through the aggregate output.

PROPOSITION 4: *In a Diamond (1965) overlapping generations economy with pollution from both consumption and production, the planner's steady state can be implemented at any given period t by the following period-by-period balanced-budget policy: Announce at $t-1$ that the following consumption tax rate, production tax rates and lump-sum transfers will apply,*

(1) to generation born at $t-1$

$$\tau^{t-1} = \frac{\gamma+\beta-[\gamma+\beta(1-b)]\,b}{\gamma b},$$

$$\tau_p^t = \frac{\alpha(\gamma+\beta)-\beta b^2(\gamma-\alpha)-\alpha\beta b}{\gamma\,[\gamma+\beta(1-b)]},$$

$$T_0^{t-1} = \frac{1}{A}\begin{pmatrix}\gamma\\ 1-b\\ -1\end{pmatrix}' \begin{pmatrix} F_L(k^{t-2},1)-(1+\tau)c_0^*-k^* \\ (1-b)E_{t-2}-\alpha F(k^{t-2},1)-\beta((1+\tau)c_0^*+c_1^{t-2})+\gamma m^{t-2} \\ E^*+\alpha F(k^*,1)+\beta(c_0^*+c_1^*)\end{pmatrix},$$

$$T_1^{t-1} = (1+\tau)c_1^* - (1-\tau_p)F_K(k^*,1)k^*,$$

where $A = \gamma+\beta(1-b)$,

(2) to all generations born from period t onwards

$$\tau = \frac{\gamma+\beta-[\gamma+\beta(1-b)]\,b}{\gamma b},$$

$$\tau_p = \frac{\alpha(\gamma+\beta)-\beta b^2(\gamma-\alpha)-\alpha\beta b}{\gamma\,[\gamma+\beta(1-b)]},$$

$$T_0 = (1-\tau_p)F_L(k^*,1)-(1+\tau)c_0^*-k^*-m^*,$$

$$T_1 = (1+\tau)c_1^*-(1-\tau_p)F_K(k^*,1)k^*.$$

Proof: The proof for this proposition is similar to the proof for Proposition 2. ■

5.4. Taxes on Consumption, Production, and Labor Income

We now modify the tax and transfer policy introduced in Section 5.3 by using the labor income tax rate τ_w^t to replace the lump-sum tax T_0^t. All other things are kept the same as in the Section 5.3 Under this policy, the agent t's problem is

$$\max_{c_0^t, c_1^t, k^t, m^t \geq 0 E_t, E_{t+1}^e} u(c_0^t) + v(c_1^t) + \phi(E_{t+1}^e) \tag{52}$$

subject to

$$(1+\tau^t)c_0^t + k^t + m^t = (1-\tau_w^t)w_t, \tag{53}$$

$$(1+\tau^t)c_1^t = r_{t+1}k^t + T_1^t, \tag{54}$$

$$E_t = (1-b)E_t - \alpha F(k^{t-1}, 1) - \beta(c_0^t + c_1^{t-1}) + \gamma m^{t-1}, \tag{55}$$

$$E_{t+1}^e = (1-b)E_t - \alpha F(K_{t+1}, 1) - \beta(c_0^{t+1,e} + c_1^t) + \gamma m^t, \tag{56}$$

given E_{t-1}, c_1^{t-1}, k^{t-1}, m^{t-1}, w_t, $c_0^{t+1,e}$, and r_{t+1}. Note again that in (56), the agent, being negligible within his generation, ignores the fact that $K_{t+1} = k^t$ and hence is unable to internalize the effect of the savings decisions on environment through the aggregate output.

PROPOSITION 5: *In a Diamond (1965) overlapping generations economy with pollution from both consumption and production, the planner's steady state can be implemented at any given period t by the following period-by-period balanced-budget policy: Announce at $t-1$ that the following consumption tax rate, production tax rate, labor tax rate and lump-sum transfer will apply,*

(1) to generation born at $t-1$

$$\tau^{t-1} = \frac{\gamma + \beta - [\gamma + \beta(1-b)]\, b}{\gamma b},$$

$$\tau_p^t = \frac{\alpha(\gamma+\beta) - \beta b^2(\gamma - \alpha) - \alpha\beta b}{\gamma\,[\gamma + \beta(1-b)]},$$

$$\tau_w^{t-1} = \frac{1}{B}\begin{pmatrix}\gamma \\ 1-b \\ -1\end{pmatrix}' \begin{pmatrix} F_L(k^{t-2}, 1) - (1+\tau)c_0^* - k^* \\ (1-b)E_{t-2} - \alpha F(k^{t-2}, 1) - \beta((1+\tau)c_0^* + c_1^{t-2}) + \gamma m^{t-2} \\ E^* + \alpha F(k^*, 1) + \beta(c_0^* + c_1^*) \end{pmatrix},$$

$$T_1^{t-1} = (1+\tau)c_1^* - (1-\tau_p)F_K(k^*, 1)k^*,$$

where $B = [\gamma + \beta(1-b)]\, F_L(k^{t-2}, 1)$,[5]

(2) *to all generations born from period* t *onwards*

$$\tau = \frac{\gamma + \beta - [\gamma + \beta(1-b)]\, b}{\gamma b},$$

$$\tau_p = \frac{\alpha(\gamma+\beta) - \beta b^2(\gamma - \alpha) - \alpha\beta b}{\gamma\,[\gamma + \beta(1-b)]},$$

$$\tau_w = 1 - \frac{(1+\tau)c_0^* + k^* + m^*}{(1-\tau_p)F_L(k^*, 1)},$$

$$T_1 = (1+\tau)c_1^* - (1-\tau_p)F_K(k^*, 1)k^*.$$

Proof: The proof for this proposition is similar to the proof for Proposition 2. ■

6. Discussion of Policies

The four alternative tax and transfer policies analyzed above conduct the economy to the same outcome, the social optimum, through the same Pareto-improving path. So from a welfare point of view, the agents are indifferent between the four policies. However, the agents pay lower taxes and/or receive higher transfers under some of the policies compared to others, which may make the former policies likelier to be voted for and implemented. In our setting, the tax and transfer policy is announced at the beginning of period $t-1$. Therefore, we will analyze the taxes and transfers that agents born in period $t-1$ pay and receive, respectively, as well as those for agents born from period t onwards.

For convenience of notation, we shall denote by subscripts 1, 2, 3, and 4, respectively, the policies "taxes on consumptions," "taxes on consumptions and capital income," "taxes on consumptions and production," and "taxes on consumptions, production, and labor income."

The following proposition summarizes how the policies compare in terms of the net taxes and transfer they generate from period $t-1$ onwards:

PROPOSITION 6: *In a Diamond (1965) overlapping generations economy with pollution from both consumption and production, if at period* $t-1$ *one of the tax and transfer policies is announced, then*

(1) *in period* $t-1$, *young agents pay the same amount of taxes under the four policies above. Hence, old agents in period* $t-1$ *receive the same amount of transfers under these policies.*

---

[5] Note that this denominator differs from the corresponding denominator in Proposition 4 by a factor $F_L(k^{t-2}, 1)$. This is obvious since τ_w^{t-1} is the tax rate on labor income while T_0^{t-1} is a lump-sum tax on income.

(2) *from period t onwards, all agents are equally treated under the policies* 1 *and* 2*; and they are also equally treated under the policies* 3 *and* 4*, specifically*

(2a) *If* $\alpha(\gamma+\beta)-\beta b^2(\gamma-\alpha)-\alpha\beta b>(<)0$*, so that* $\tau_p>(<)0$*, then they pay less (more) taxes when young and receive more (less) net transfers when old with policies* 3 *and* 4 *than with the two other.*

(2b) *If* $\alpha(\gamma+\beta)-\beta b^2(\gamma-\alpha)-\alpha\beta b=0$*, so that* $\tau_p=0$ *and* $\tau_k=0$*, then policies* 1, 2, *and* 3 *coincide, and agents are equally treated by all four policies.*

Proof: See Appendix A.6 ■

According to Proposition 6, which policy would be voted for by the agents depends on the condition guaranteeing the production tax rate τ_p to be positive, or negative, or zero. When $\alpha(\gamma+\beta)-\beta b^2(\gamma-\alpha)-\alpha\beta b>0$, Proposition 6 shows that policies 3 and 4 will likely be preferred by the agents since they pay directly less taxes when young and receive more net transfer when old with these policies than with the others. That is because under policies 3 and 4, firms pay part of the taxes while under the others they do not, and agents may not perceive taxes paid by the firm as undistributed income. In fact, when firms pay taxes, wages decrease. Actually, each agent being negligible takes into account only how taxes paid directly affect his/her budget constraints, disregarding how taxes paid by firms affect his/her budget constraints. Therefore, they will likely vote for policies 3 and 4. Now, from the policy maker's point of view, the only difference between policies 3 and 4 is the labor income tax versus the lump-sum tax raised on the income of the young agents. For the same amount of tax, the labor income tax seems more easily implementable because of being proportional on the agents' wages while the policy maker may have to set a heterogeneous lump-sum tax applying to heterogeneous agents in reality.

In the case $\alpha(\gamma+\beta)-\beta b^2(\gamma-\alpha)-\alpha\beta b<0$, agents may prefer policies 1 and 2, because under policies 3 and 4 agents pay a higher tax when young and receive a lower transfer when old to subsidize production. Indeed, being negligible, agents cannot internalize that their paid tax subsidize production that will increase wages making the policies equivalent. From the policy maker's point of view, policy 2 may be more easily implementable than policy 1, since policy 1 requires discriminating between young and old consumers about tax rates on consumption, the implementation of which may be costly. For policy 2, however, the policy maker applies a single consumption tax rate to both young and old, along with a capital income tax on the returns to savings lent to firms via the banking system, and hence easy to collect at a low cost.

For the case $\alpha(\gamma+\beta)-\beta b^2(\gamma-\alpha)-\alpha\beta b=0$, agents are equally treated by all four policies. Moreover, under this condition the policies 1, 2, and 3 exactly coincide. In this case, as discussed above, from the policy maker's point of view policy 4 is likely to be more easily implementable than the others.

7. Conclusion

We have presented a general equilibrium overlapping generations model with environmental externalities from both production and consumption. For such a model we proved that the competitive equilibrium steady state is not the planner's steady state, for any discount rate the social planner may use. The pollution externality from consumption does not affect the range of dynamically inefficient capital ratios, whereas the pollution externality from production does. The higher the production pollution parameter α, the larger the inefficient range. The environment-maintaining technology γ also plays a role in determining the best steady state capital ratio k^*. The cleaner the environment-maintaining technology, the smaller the range of the dynamically inefficient allocations. By comparing the competitive steady state and the best steady state, we designed balanced budget taxes and transfer policies that decentralize the planner's steady state. We also discussed the desirability and implementability of each policy from the viewpoints of both the agents and the policy maker.

This paper makes many simplifying assumptions, such as the technology being exogenous, the population growth rate being zero, and there being only one production sector. Further developments, including endogenous technology and fertility, as well as the impact of human capital accumulation, are left for future research.

Appendix

A.1. Existence of the Agent's Optimal Solution

By substituting (11), (12), (13), and (14) into (9) and (10) the existence of solution to the system of the first-order conditions (9)–(14) is equivalent to the existence of solution to the system of two following equations:

$$u'(c_0^t) - [\beta(1-b)+\gamma]\,\phi'(E_{t+1}^e) = 0, \tag{A1}$$

$$v'(c_1^t) - \frac{\beta + \frac{\gamma}{r_{t+1}}}{\beta(1-b)+\gamma}\,u'(c_0^t) = 0, \tag{A2}$$

where

$$\begin{aligned} E_{t+1}^e = (1-b)\left[(1-b)E_{t-1} - \alpha F(k_{t-1},1) - \beta(c_0^t + c_1^{t-1}) + \gamma m^{t-1}\right] \\ -\alpha F(K_{t+1},1) - \beta(c_0^{t+1,e} + c_1^t) + \gamma(w_t - c_0^t - \tfrac{c_1^t}{r_{t+1}}). \end{aligned}$$

From (A2), by the implicit function theorem we can treat c_1^t as a function of c_0^t, $c_1^t = \varphi(c_0^t)$ where $\varphi'(\cdot) > 0$, $\varphi(0) = 0$ and $\varphi(+\infty) = +\infty$. We rewrite

$$E_{t+1}^e = Q - [\beta(1-b) + \gamma]\, c_0^t - \left(\beta + \frac{\gamma}{r_{t+1}}\right)\varphi(c_0^t),$$

where $Q = (1-b)\left[(1-b)E_{t-1} - \alpha F(k_{t-1}, 1) - \beta c_1^{t-1} + \gamma e^{t-1}\right] - \alpha F(K_{t+1}, 1) - \beta c_0^{t+1,e} + \gamma w_t$. Now the system of Equations (A1) and (A2) leads to the following equation:

$$u'(c_0^t) - [\beta(1-b) + \gamma]\,\phi'\ \Big(Q - [\beta(1-b) + \gamma]\, c_0^t - \left(\beta + \frac{\gamma}{r_{t+1}}\right)\varphi(c_0^t)\Big) = 0. \tag{A3}$$

The existence of the agent's optimal solution is equivalent to the existence of solution to Equation (A3). In effect, set

$$\triangle = u'(c_0^t) - [\beta(1-b) + \gamma]\,\phi'\left(Q - [\beta(1-b) + \gamma]\, c_0^t - \left(\beta + \frac{\gamma}{r_{t+1}}\right)\varphi(c_0^t)\right)$$

is a continuous function of c_0^t. We have,

$$\lim_{c_0^t \to +\infty} \triangle = -\infty < 0$$

and

$$\lim_{c_0^t \to 0^+} \triangle = +\infty > 0.$$

We also find that $\triangle$ is a monotone function of c_0^t since

$$\frac{\partial \triangle}{\partial c_0^t} = u''(c_0^t) + [\beta(1-b) + \gamma]\left[\beta(1-b) + \gamma + \left(\beta + \frac{\gamma}{r_{t+1}}\right)\varphi'(c_0^t)\right]\phi''(E_{t+1}^e) < 0.$$

So there exists a unique solution $c_0^t > 0$ to (A3), meaning that there exists a unique optimal solution of the agent.

A.2. Checking the SOCs for the Maximization Problem of the Agent

For the FOCs to be sufficient conditions to characterize a (local) maximum to the optimization problem, we have to check the sufficient SOCs. The Lagrangian of the maximization problem is

$$Z_t = u(c_0^t) + v(c_1^t) + \phi(E_{t+1}^e) + \lambda_1^t(c_0^t + k^t + m^t - w_t) + \lambda_2^t(c_1^t - r_{t+1}k^t) + \lambda_3^t\left(E_t - (1-b)E_{t-1} + \alpha F(k^{t-1}, 1) + \beta(c_0^t + c_1^{t-1}) - \gamma m^{t-1}\right)$$

$$+\lambda_4^t \left(E_{t+1}^e - (1-b) E_t + \alpha F(k^t, 1) + \beta (c_0^{t+1,e} + c_1^t) - \gamma m^t \right),$$

whose bordered Hessian will appear as

$$\bar{H} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -r_{t+1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta & 0 & -\gamma & b-1 & 1 \\ 1 & 0 & \beta & 0 & u''(c_0^t) & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \beta & 0 & v''(c_1^t) & 0 & 0 & 0 & 0 \\ 1 & -r_{t+1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -\gamma & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & b-1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \phi''(E_{t+1}^e) \end{pmatrix}.$$

The sufficient SOCs for a maximum are

$$(-1)^5 \left|\bar{H}_5\right| = -\begin{vmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -r_{t+1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \beta & 0 & -\gamma & b-1 \\ 1 & 0 & \beta & 0 & u''(c_0^t) & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \beta & 0 & v''(c_1^t) & 0 & 0 & 0 \\ 1 & -r_{t+1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -\gamma & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & b-1 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

$$= -\left(\beta(1-b) + \gamma\right)^2 r_{t+1}^2 v''(c_1^t) - \left(\beta r_{t+1} + \gamma\right)^2 u''(c_0^t) > 0,$$

$$(-1)^6 \left|\bar{H}_6\right| = \begin{vmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -r_{t+1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta & 0 & -\gamma & b-1 & 1 \\ 1 & 0 & \beta & 0 & u''(c_0^t) & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \beta & 0 & v''(c_1^t) & 0 & 0 & 0 & 0 \\ 1 & -r_{t+1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -\gamma & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & b-1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \phi''(E_{t+1}^e) \end{vmatrix}$$

$$= r_{t+1}^2 v''(c_1^t) u''(c_0^t) + \phi''(E_{t+1}^e) \left|\bar{H}_5\right| > 0,$$

which guarantees that the solution to the agent's problem is a maximum indeed.

A.3. Competitive Equilibrium Dynamics

The competitive equilibrium conditions impose on $(c_0^{t+1}, c_1^t, k^t, m^t, E_{t+1})$ a dynamics described by a first-order difference equation, because

$$u'(c_0^t) - [\beta(1-b) + \gamma]\,\phi'(E_{t+1}) = 0,$$

$$v'(c_1^t) - \left[\beta + \frac{\gamma}{F_K(k^t,1)}\right]\phi'(E_{t+1}) = 0,$$

$$c_0^t + k^t + m^t - F_L(k^{t-1},1) = 0,$$

$$c_1^t - F_K(k^t,1)k^t = 0,$$

$$E_{t+1} - (1-b)E_t + \alpha F(k^t,1) + \beta(c_0^{t+1} + c_1^t) - \gamma m^t = 0,$$

implicitly define it to be a function of its lagged value $(c_0^t, c_1^{t-1}, k^{t-1}, m^{t-1}, E_t)$. In effect, the associated Jacobian matrix with respect to $(c_0^{t+1}, c_1^t, k^t, m^t, E_{t+1})$

$$J = \begin{pmatrix} 0 & 0 & 0 & 0 & G \\ 0 & v''(c_1^t) & D & 0 & H \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & -C & 0 & 0 \\ \beta & \beta & \alpha F_K(k^t,1) & -\gamma & 1 \end{pmatrix},$$

where

$$C = F_K(k^t,1) + F_{KK}(k^t,1)k^t > 0,$$

$$D = \frac{\gamma F_{KK}(k^t,1)}{F_K(k^t,1)^2}\phi'(E_{t+1}) < 0,$$

$$G = -[\beta(1-b)+\gamma]\,\phi''(E_{t+1}) > 0,$$

$$H = -\left[\beta + \frac{\gamma}{F_K(k^t,1)}\right]\phi''(E_{t+1}) > 0,$$

is regular, since

$$\det(J) = G\begin{vmatrix} 0 & v''(c_1^t) & D & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & -C & 0 \\ \beta & \beta & \alpha F_K(k^t,1) & -\gamma \end{vmatrix}$$

$$= -G\beta\begin{vmatrix} v''(c_1^t) & D & 0 \\ 0 & 1 & 1 \\ 1 & -C & 0 \end{vmatrix}$$

$$= -G\beta\left[D + Cv''(c_1^t)\right] > 0.$$

Because the Jacobian matrix is regular for all $(c_0^{t+1}, c_1^t, k^t, m^t, E_{t+1})$, it is evidently regular at the solution. This implies that for all competitive equilibrium $(c_0^{t+1}, c_1^t, k^t, m^t, E_{t+1})_t$ there exists, for all t, a function $\psi : \mathbb{R}^5 \to \mathbb{R}^5$ such that

$$\begin{pmatrix} c_0^{t+1} \\ c_1^t \\ k^t \\ m^t \\ E_{t+1} \end{pmatrix} = \psi \begin{pmatrix} c_0^t \\ c_1^{t-1} \\ k^{t-1} \\ m^{t-1} \\ E_t \end{pmatrix}.$$

A.4. Solving the Problem of the Social Planner

The Lagrange function for this problem is

$$\mathcal{L} = \sum_{t=0}^{+\infty} \frac{1}{(1+R)^t} \left[u(c_0^t) + u(c_1^t) + \phi(E_{t+1})\right]$$
$$+ \sum_{t=0}^{+\infty} \frac{\mu_t}{(1+R)^t} \left[F(k^{t-1}, 1) - c_0^t - c_1^{t-1} - k^t - m^t\right]$$
$$+ \sum_{t=0}^{+\infty} \frac{\eta_t}{(1+R)^t} \left[E_{t+1} - (1-b)E_t + \alpha F(k^t, 1) + \beta(c_0^{t+1} + c_1^t) - \gamma m^t\right].$$

The FOCs of the maximization problem are

$$\frac{\partial \mathcal{L}}{\partial c_0^t} = \frac{u'(c_0^t)}{(1+R)^t} - \frac{\mu_t}{(1+R)^t} + \frac{\beta \eta_{t-1}}{(1+R)^{t-1}} = 0,$$
$$\frac{\partial \mathcal{L}}{\partial c_1^t} = \frac{v'(c_1^t)}{(1+R)^t} - \frac{\mu_{t+1}}{(1+R)^{t+1}} + \frac{\beta \eta_t}{(1+R)^t} = 0,$$
$$\frac{\partial \mathcal{L}}{\partial E_{t+1}} = \frac{\phi'(E_{t+1})}{(1+R)^t} + \frac{\eta_t}{(1+R)^t} - \frac{\eta_{t+1}(1-b)}{(1+R)^{t+1}} = 0,$$
$$\frac{\partial \mathcal{L}}{\partial k^t} = -\frac{\mu_t}{(1+R)^t} + \frac{\mu_{t+1} F_K(k^t, 1)}{(1+R)^{t+1}} + \frac{\eta_t \alpha F_K(k^t, 1)}{(1+R)^t} = 0,$$
$$\frac{\partial \mathcal{L}}{\partial m^t} = -\frac{\mu_t}{(1+R)^t} - \frac{\eta_t \gamma}{(1+R)^t} = 0,$$

that is to say

$$u'(c_0^t) - \mu_t + \beta \eta_{t-1}(1+R) = 0,$$
$$v'(c_1^t) - \frac{\mu_{t+1}}{1+R} + \beta \eta_t = 0,$$
$$\phi'(E_{t+1}) + \eta_t - \frac{\eta_{t+1}(1-b)}{1+R} = 0,$$

$$-\mu_t + \frac{\mu_{t+1} F_K(k^t, 1)}{1+R} + \eta_t \alpha F_K(k^t, 1) = 0,$$
$$-\mu_t - \eta_t \gamma = 0.$$

At the steady state,

$$u'(\bar{c}_0) = \mu - \beta\eta(1+R),$$
$$v'(\bar{c}_1) = \frac{\mu}{1+R} - \beta\eta,$$
$$\phi'(\bar{E}) = -\eta + \frac{(1-b)\eta}{1+R},$$
$$F_K(\bar{k}, 1) = \frac{\mu(1+R)}{\mu + \alpha\eta(1+R)},$$
$$\mu = -\eta\gamma.$$

Therefore,

$$u'(\bar{c}_0) = (1+R)\frac{\gamma + \beta(1+R)}{b+R}\phi'(\bar{E}),$$
$$v'(\bar{c}_1) = \frac{\gamma + \beta(1+R)}{b+R}\phi'(\bar{E}),$$
$$F_K(\bar{k}, 1) = \frac{\gamma(1+R)}{\gamma - (1+R)\alpha}.$$

A.5. Proof of Proposition 2

Note first that, since a competitive equilibrium steady state under a period-by-period balanced-budget stationary policy of consumption tax rates τ_0, τ_1, and lump-sum transfers T_0, T_1 is characterized by

$$u'(c_0) = [\beta(1-b) + \gamma(1+\tau_0)]\,\phi'(E),$$
$$u'(c_0) = \left[\beta + \frac{\gamma}{F_K(k,1)}(1+\tau_1)\right]\phi'(E),$$
$$(1+\tau_0)c_0 + k + m = F_L(k, 1) - T_0,$$
$$(1+\tau_1)c_1 = F_K(k, 1)k + T_1,$$
$$E = (1-b)E - \alpha F(k, 1) - \beta(c_0 + c_1) + \gamma m,$$

and

$$\tau_0 c_0 + \tau_1 c_1 + T_0 = T_1,$$

and because the planner's steady state is characterized by

$$u'(c_0^*) = \frac{\gamma + \beta}{b}\phi'(E^*),$$

$$v'(c_1^*) = \frac{\gamma + \beta}{b}\phi'(E^*),$$

$$F_K(k^*, 1) = \frac{\gamma}{\gamma - \alpha},$$

$$c_0^* + c_1^* + k^* + m^* = F(k^*, 1),$$

$$E^* = (1 - b)E^* - \alpha F(k^*, 1) - \beta(c_0^* + c_1^*) + \gamma m^*,$$

then for the stationary competitive equilibrium under τ_0, τ_1, T_0, T_1 to be the planner's steady state it is necessary and sufficient that

$$\beta(1 - b) + \gamma(1 + \tau_0) = \frac{\gamma + \beta}{b},$$

$$\beta + \frac{\gamma}{F_K(k^*, 1)}(1 + \tau_1) = \frac{\gamma + \beta}{b},$$

$$(1 + \tau_0)c_0^* + k^* + m^* = F_L(k^*, 1) - T_0,$$

$$(1 + \tau_1)c_1^* = F_K(k^*, 1)k^* + T_1,$$

i.e., that

$$\tau_0 = \frac{\gamma + \beta - [\gamma + \beta(1 - b)]\, b}{\gamma b},$$

$$\tau_1 = \frac{\gamma + \beta(1 - b)}{(\gamma - \alpha)b} - 1,$$

$$T_0 = F_L(k^*, 1) - (1 + \tau_0)c_0^* - k^* - m^*,$$

$$T_1 = (1 + \tau_1)c_1^* - F_K(k^*, 1)k^*.$$

In other words, such a policy supports the planner's steady state *once the economy is there.* There is then the additional problem of *moving the economy to the planner's steady state* .

To address this additional problem, it should be noticed that the choice by agent t of c_0^t, c_1^t, k^t, m^t, E_t, E_{t+1}^e depends on past decisions, in particular on k^{t-1} through $w_t = F_L(k^{t-1}, 1)$, and on c_1^{t-1}, m^{t-1}, E_{t-1} through the environment dynamics. We shall show next that there exists a balanced-budget policy toward generation $t - 1$ that makes it choose c_0^*, c_1^*, k^* and $E_t^e = E^*$ (but not m^* or $E_{t-1} = E^*$), and another policy towards generation t, namely the policy above, that makes it choose c_0^*, c_1^*, k^* and $E_{t+1}^e = E^*$ as well as m^* and $E_t = E^*$. Once there the same policy keeps the economy at the planner's steady state.

In effect, agent $t - 1$'s choice at a perfect foresight equilibrium is characterized by the conditions

$$u'(c_0^{t-1}) = [\beta(1 - b) + \gamma(1 + \tau_0)]\phi'(E_t),$$

$$v'(c_1^{t-1}) = [\beta + \frac{\gamma}{F_K(k^{t-1},1)}(1+\tau_1)]\phi'(E_t),$$

$$(1+\tau_0)c_0^{t-1} + k^{t-1} + m^{t-1} = F_L(k^{t-2},1) - T_0^{t-1},$$

$$(1+\tau_1)c_1^{t-1} = F_K(k^{t-1},1)k^{t-1} + T_1^{t-1},$$

$$E_{t-1} = (1-b)E_{t-2} - \alpha F(k^{t-2},1) - \beta(c_0^{t-1} + c_1^{t-2} + T_1^{t-2}) + \gamma m^{t-2},$$

$$E_t = (1-b)E_{t-1} - \alpha F(k^{t-1},1) - \beta(c_0^t + c_1^{t-1}) + \gamma m^{t-1},$$

given past decisions c_1^{t-2}, k^{t-2}, m^{t-2}, E_{t-2}, and c_0^t. Note that, for the government's budget to be balanced at $t-1$, generation $t-2$ receives as a transfer T_1^{t-2} all the taxes raised at $t-1$, i.e.,

$$T_1^{t-2} = \tau_0 c_0^{t-1} + T_0^{t-1}.$$

Keeping this in mind, it turns out that there exist transfers T_0^{t-1}, T_1^{t-1} such that under τ_0, τ_1 above generation $t-1$ chooses c_0^*, c_1^*, k^* and $E_t^e = E^*$. In effect, there is a solution in c_0^{t-1}, T_0^{t-1}, T_1^{t-1}, m^{t-1}, E_{t-1}, E_t to the system above with $c^{t-1} = c_1^*$ and $k^{t-1} = k^*$, i.e., a solution to

$$u'(c_0^{t-1}) = [\beta(1-b) + \gamma(1+\tau_0)]\phi'(E_t),$$

$$v'(c_1^*) = [\beta + \frac{\gamma}{F_K(k^*,1)}(1+\tau_1)]\phi'(E_t),$$

$$(1+\tau_0)c_0^{t-1} + k^* + m^{t-1} = F_L(k^{t-2},1) - T_0^{t-1},$$

$$(1+\tau_1)c_1^* = F_K(k^*,1)k^* + T_1^{t-1},$$

$$E_{t-1} = (1-b)E_{t-2} - \alpha F(k^{t-2},1) - \beta((1+\tau_0)c_0^{t-1} + c_1^{t-2} + T_0^{t-1}) + \gamma m^{t-2},$$

$$E_t = (1-b)E_{t-1} - \alpha F(k^*,1) - \beta(c_0^t + c_1^*) + \gamma m^{t-1},$$

(where c_1^{t-1} and k^{t-1} have been fixed at the levels c_1^* and k^*, respectively) because, given the conditions characterizing the planner's steady state, from the second equation necessarily $E_t = E^*$, which in turn implies, from the first equation, that $c_0^{t-1} = c_0^*$. The fourth equation directly determines

$$T_1^{t-1} = (1+\tau_1)c_1^* - F_K(k^*,1)k^*$$

and the three other equations constitute the following regular linear system in m^{t-1}, T_0^{t-1}, and E_{t-1}

$$T_0^{t-1} + m^{t-1} = F_L(k^{t-2},1) - (1+\tau_0)c_0^* - k^*,$$

$$E_{t-1} + \beta T_0^{t-1} = (1-b)E_{t-2} - \alpha F(k^{t-2},1) - \beta((1+\tau_0)c_0^* + c_1^{t-2}) + \gamma m^{t-2},$$

$$(1-b)E_{t-1} + \gamma m^{t-1} = E_t + \alpha F(k^*,1) + \beta(c_0^t + c_1^*),$$

with solution

$$\begin{pmatrix} m^{t-1} \\ T_0^{t-1} \\ E_{t-1} \end{pmatrix} = \frac{1}{A}\begin{pmatrix} (1-b)\beta & b-1 & 1 \\ \gamma & 1-b & -1 \\ -\gamma\beta & \gamma & \beta \end{pmatrix}$$

$$\begin{pmatrix} F_L(k^{t-2},1) - (1+\tau_0)c_0^* - k^* \\ (1-b)E_{t-2} - \alpha F(k^{t-2},1) - \beta((1+\tau_0)c_0^* + c_1^{t-2}) + \gamma m^{t-2} \\ E_t + \alpha F(k^*,1) + \beta(c_0^t + c_1^*) \end{pmatrix},$$

where $A = \gamma + \beta(1-b)$ is the determinant of the matrix of coefficients.

Thus, under the following policy of consumption tax rates and lump-sum transfers

$$\tau_0^{t-1} = \frac{\gamma + \beta - [\gamma + \beta(1-b)]b}{\gamma b},$$

$$\tau_1^{t-1} = \frac{\gamma + \beta(1-b)}{(\gamma - \alpha)b} - 1,$$

$$T_0^{t-1} = \frac{1}{A}\begin{pmatrix} \gamma \\ 1-b \\ -1 \end{pmatrix}'$$

$$\begin{pmatrix} F_L(k^{t-2},1) - (1+\tau_0)c_0^* - k^* \\ (1-b)E_{t-2} - \alpha F(k^{t-2},1) - \beta((1+\tau_0)c_0^* + c_1^{t-2}) + \gamma m^{t-2} \\ E_t + \alpha F(k^*,1) + \beta(c_0^t + c_1^*) \end{pmatrix},$$

$$T_1^{t-1} = (1+\tau_1)c_1^* - F_K(k^*,1)k^*,$$

generation $t-1$ makes at equilibrium the choices

$$c_0^{t-1} = c_0^*,$$

$$c_1^{t-1} = c_1^*,$$

$$k^{t-1} = k^*,$$

$$m^{t-1} = \frac{1}{A}\begin{pmatrix} (1-b)\beta \\ b-1 \\ 1 \end{pmatrix}'$$

$$\begin{pmatrix} F_L(k^{t-2},1) - (1+\tau_0)c_0^* - k^* \\ (1-b)E_{t-2} - \alpha F(k^{t-2},1) - \beta((1+\tau_0)c_0^* + c_1^{t-2}) + \gamma m^{t-2} \\ E_t + \alpha F(k^*,1) + \beta(c_0^t + c_1^*) \end{pmatrix},$$

$$E_{t-1} = \frac{1}{A}\begin{pmatrix} -\gamma\beta \\ \gamma \\ \beta \end{pmatrix}' \begin{pmatrix} F_L(k^{t-2}, 1) - (1+\tau_0)c_0^* - k^* \\ (1-b)E_{t-2} - \alpha F(k^{t-2}, 1) - \beta((1+\tau_0)c_0^* + c_1^{t-2}) + \gamma m^{t-2} \\ E_t + \alpha F(k^*, 1) + \beta(c_0^t + c_1^*) \end{pmatrix}$$

$$E_t = E^*.$$

Note that the policy above depends on elements known at the time $t-1$ announces it, except for the transfer T_0^{t-1}, which depends as well on the expected (and, at a perfect foresight equilibrium, the actual) first-period consumption of generation t, c_0^t. The choice of m^{t-1} and E_{t-1} by generation $t-1$ depends on it accordingly. Nevertheless, under the announcement at $t-1$ that the same consumption tax rates will be applied to generation t as well, along with the stationary balanced-budget transfers implementing the planner's steady state, i.e., under the policy

$$\tau_0^t = \frac{\gamma + \beta - [\gamma + \beta(1-b)]b}{\gamma b},$$

$$\tau_1^t = \frac{\gamma + \beta(1-b)}{(\gamma - \alpha)b} - 1,$$

$$T_0 = F_L(k^*, 1) - (1+\tau_0)c_0^* - k^* - m^*,$$

$$T_1 = (1+\tau_1)c_1^* - F_K(k^*, 1)k^*.$$

It is perfectly foreseen at $t-1$ that at equilibrium $c_0^t = c_0^*$. More specifically, under this policy generation t's choice is c_0^*, c_1^*, k^*, m^*, $E_t = E^*$, and $E_{t+1}^e = E^*$, because this choice solves

$$u'(c_0^t) = [\beta(1-b) + \gamma(1+\tau_0)]\phi'(E_{t+1}^e),$$

$$v'(c_1^t) = [\beta + \frac{\gamma}{F_K(k^t, 1)}(1+\tau_1)]\phi'(E_{t+1}^e),$$

$$(1+\tau_0)c_0^t + k^t + m^t = F_L(k^*, 1) - T_0^t,$$

$$(1+\tau_1)c_1^t = F_K(k^t, 1)k^t + T_1^t,$$

$$E_t = (1-b)E_{t-1} - \alpha F(k^*, 1) - \beta(c_0^t + c_1^*) + \gamma m^{t-1},$$

$$E_{t+1}^e = (1-b)E_t - \alpha F(k^t, 1) - \beta(c_0^{t+1,e} + c_1^t) + \gamma m^t,$$

given that E_{t+1}^e, $c_0^{t+1,e}$ are perfectly foreseen to be E^*, c_0^* when the policy is left unchanged for all generations from generation t onwards, and the next-to-last equation is guaranteed to be satisfied from the choices of E_{t-1} and

m^{t-1} by generation $t-1$ at $t-1$ (it is the last equation in generation $t-1$'s system).

Finally, note that at period t the policy is balanced as well (at $t-1$ and from $t+1$ onwards it is so by construction), i.e.,

$$T_1^{t-1} = \tau_0 c_0^t + \tau_1 c_1^{t-1} + T_0^t,$$

that is to say,

$$(1+\tau_1)c_1^* - F_K(k^*, 1)k^* = \tau_0 c_0^* + \tau_1 c_1^* + F_L(k^*, 1) - (1+\tau_0)c_0^* - k^* - m^*,$$

which boils down to

$$c_0^* + c_1^* + k^* + m^* = F_K(k^*, 1)k^* + F_L(k^*, 1),$$

which is guaranteed by the feasibility of the planner's steady state. ■

A.6. Proof of Proposition 6

(1) It should be noticed that, under each policy above, each agent born in period $t-1$ chooses $c_0^{t-1} = c_0^*$, $c_1^{t-1} = c_1^*$, $k^{t-1} = k^*$, $E_t^e = E^*$ (but not $m^{t-1} = m^*$ and not $E_{t-1} = E^*$). So, young agents pay the following amounts Γ_i^{t-1} of taxes under each policy i[6]

Policy 1

$$\Gamma_1^{t-1} = \tau_0 c_0^* + T_0^{t-1}.$$

Policy 2

$$\Gamma_2^{t-1} = \tau c_0^* + T_0^{t-1}.$$

Policy 3

$$\Gamma_3^{t-1} = \tau c_0^* + T_0^{t-1}.$$

Policy 4

$$\Gamma_4^{t-1} = \tau c_0^* + \tau_w^{t-1} F_L(k^{t-2}, 1).$$

From the equations determining tax rates and lump-sum taxes and transfers in Propositions 2–5, we know that $\tau_0 = \tau$, and the lump-sum taxes imposed on the incomes of agents born in period $t-1$ in policies 1, 2, and 3 are exactly identical. And it is straightforward from equation defining τ_w^{t-1} (in Proposition 5) to find that $\tau_w^{t-1} F_L(k^{t-2}, 1) = T_0^{t-1}$. Therefore, we have

$$\Gamma_1^{t-1} = \Gamma_2^{t-1} = \Gamma_3^{t-1} = \Gamma_4^{t-1},$$

[6] Since the tax rates are unchanged over time, except for the tax rate on labor income, we have removed the time superscripts for all tax rates, but not for the tax rate on labor income in period $t-1$.

i.e., the agents born in period $t-1$ pay the same amount of taxes when young under each of the policies above.
(2) First of all, we prove that from period t onwards, agents are treated equally under policies 1 and 2, and they are also treated equally under policies 3 and 4. It should be noted that from period t onwards, under each policy above, agents choose $c_0^{t+i} = c_0^*$, $c_1^{t+i} = c_1^*$, $k^{t+i} = k^*$, $m^{t+i} = m^*$, $E_{t+i} = E^*$, and $E^e_{t+i+1} = E^* \forall i \geq 0$. So, from period t onwards, each young agent has to pay an amount of taxes corresponding to each alternative policy as follows:[7]

Policy 1

$$\Gamma_1 = \tau_0 c_0^* + T_{0,1} = F_L(k^*, 1) - c_0^* - k^* - m^*.$$

Policy 2

$$\Gamma_2 = \tau c_0^* + T_{0,2} = F_L(k^*, 1) - c_0^* - k^* - m^*.$$

Policy 3

$$\Gamma_3 = \tau c_0^* + T_{0,3} = (1-\tau_p) F_L(k^*, 1) - c_0^* - k^* - m^*.$$

Policy 4

$$\Gamma_4 = \tau c_0^* + \tau_w F_L(k^*, 1) = (1-\tau_p) F_L(k^*, 1) - c_0^* - k^* - m^*.$$

Hence, we have

$$\Gamma_1 = \Gamma_2 \qquad \text{and} \qquad \Gamma_3 = \Gamma_4. \tag{A4}$$

Each old agent from period t onwards receives an amount of net transfers (lump-sum transfer after subtracting taxes paid) corresponding to each alternative policy as follows:

Policy 1

$$\Theta_1 = T_{1,1} - \tau_1 c_1^* = c_1^* - F_K(k^*, 1) k^*.$$

Policy 2

$$\Theta_2 = T_{1,2} - \tau c_1^* - \tau_k F_K(k^*, 1) k^* = c_1^* - F_K(k^*, 1) k^*.$$

Policy 3

$$\Theta_3 = T_{1,3} - \tau c_1^* = c_1^* - (1-\tau_p) F_K(k^*, 1) k^*.$$

Policy 4

$$\Theta_4 = T_{1,4} - \tau c_1^* = c_1^* - (1-\tau_p) F_K(k^*, 1) k^*.$$

Hence, we have

$$\Theta_1 = \Theta_2 \qquad \textit{and} \qquad \Theta_3 = \Theta_4. \tag{A5}$$

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[7] In this proof, we add subscripts 1, 2, 3, and 4 into the corresponding lump-sum taxes and lump-sum transfers to indicate that they belong to corresponding policy 1, 2, 3, or 4.

(A4) and (A5) imply that agents are treated equally under policies 1 and 2, and they are also treated equally under policies 3 and 4.

(2a) If $\alpha(\gamma+\beta)-\beta b^2(\gamma-\alpha)-\alpha\beta b>(<)0$, then $\tau_k, \tau_p>(<)0$, and therefore

$$\Gamma_1=\Gamma_2>(<)\Gamma_3=\Gamma_4$$

and

$$\Theta_1=\Theta_2<(>)\Theta_3=\Theta_4,$$

which imply (2a) of Proposition 6.

(2b) If $\alpha(\gamma+\beta)-\beta b^2(\gamma-\alpha)-\alpha\beta b=0$, then $\tau_k, \tau_p=0$, and therefore

$$\Gamma_1=\Gamma_2=\Gamma_3=\Gamma_4$$

and

$$\Theta_1=\Theta_2=\Theta_3=\Theta_4,$$

i.e., in this case agents will be equally treated under all policies.

We prove next that, in this case policies 1, 2, and 3 coincide. In effect, from equations determining the taxes and transfers in Propositions 3 and 4, it is straightforward that policies 2 and 3 coincide. By comparing equations in Proposition 3 (or Proposition 4) with equations in Proposition 2, we now complete the proof by showing that under the condition $\alpha(\gamma+\beta)-\beta b^2(\gamma-\alpha)-\alpha\beta b=0$ the tax rates τ_0 and τ_1, which are determined in Proposition 2, are equal.

We have

$$\alpha(\gamma+\beta)-\beta b^2(\gamma-\alpha)-\alpha\beta b=0$$
$$\Leftrightarrow \quad \gamma+\beta=\frac{\beta b^2\gamma}{\alpha}-\beta b^2+\beta b.$$

We transform τ_0 and τ_1 as follows:

$$\tau_0=\frac{\gamma+\beta-[\gamma+\beta(1-b)]\,b}{\gamma b}$$
$$=\frac{\frac{\beta b^2\gamma}{\alpha}-\beta b^2+\beta b-[\gamma+\beta(1-b)]\,b}{\gamma b}=\frac{\beta b}{\alpha}-1$$

and

$$\tau_1=\frac{\gamma+\beta(1-b)}{(\gamma-\alpha)b}-1$$
$$=\frac{\frac{\beta b^2\gamma}{\alpha}-\beta b^2+\beta b-\beta b}{(\gamma-\alpha)b}-1=\frac{\beta b}{\alpha}-1.$$

Therefore, $\tau_0 = \tau_1 = \tau$. Hence, policy 1 also coincides with policies 2 and 3. ■

References

DIAMOND, P. A. (1965) National debt in a neoclassical growth model, *American Economic Review* **55**, 1126–1150.

GUTIÉRREZ, M. J. (2008) Dynamic inefficiency in an overlapping generation economy with pollution and health cost, *Journal of Public Economic Theory* **10**, 563–594.

JOHN, A., and R. PECCHENINO (1994) An overlapping generations model of growth and the environment, *Economic Journal* **104**, 1393–1410.

JOHN, A., R. PECCHENINO, D. SCHIMMELPFENNIG, and S. SCHREIT (1995) Short-lived agents and the long-lived environment, *Journal of Public Economics* **58**, 127–141.

JOUVET, P. A., P. H. MICHEL, and J. P. VIDAL (2000) Intergenerational altruism and the environment, *Scandinavian Journal of Economics* **102**, 135–150.

JOUVET, P. A., P. PESTIEAU, and G. PONTHIERE (2010) Longevity and environmental quality in an OLG model, *Journal of Economics* **100**, 191–216.

MARINI, G., and P. SCARAMOZZINO (1995) Overlapping generations and environmental control, *Journal of Environmental Economics and Management* **29**(1), 64–77.

ONO, T. (1996) Optimal tax schemes and the environmental externality, *Economics Letters* **53**, 283–289.

PAUTREL, X. (2009) Pollution and life expectancy: How environmental policy can prompt growth, *Ecological Economics* **68**, 1040–1051.