Education Technology and Mathematics

Miles Berry

November 2021

Digital technology has much potential to support, extend, enhance and enrich school mathematics education. At present, the most of its use in UK schools seems focused on making traditional pedagogic approaches, such as exposition, worked examples and repeated practice more efficient, and yet in the 'real world' beyond school, the applications of mathematics, and the experience of *doing* mathematics have been totally transformed by the use of computers to, well, compute. There is a long history of attempts to integrate digital technology into the nature of school mathematics as a subject, but it is in only a minority of schools where this happens today. Wolfram (2020) and others have argued for a fundamental transformation in the nature of school mathematics, focussing far more on the problem solving steps of definition, abstraction and interpretation, whilst leaving the dull, mechanical computation to the machines - will such an approach characterize mathematical futures in schools, or will the role of digital technology continue to be limited to little more than supporting (or replacing) teachers in their tasks of setting and marking work?

The discussion below seeks to summarise the present position, based mostly on anecdotal evidence from primary and secondary teachers rather than any empirical studies. I offer some suggestions, rather than predictions, for the role of digital technology in mathematical futures, and suggest some questions which phase 2 of the Royal Society's project might usefully pursue in this area.

The potential of digital technologies

What does digital technology offer that older technologies do not? Loveless (2002) identified six characteristics of digital technology, "provisionality, interactivity, capacity, range, speed and automatic functions" (p3), arguing that these enabled users to do things "that could not be done as effectively, or at all, using other tools" (op. cit.). All of these characteristics have direct application to mathematics education. Thus, for example, provisionality allows pupils to try ideas out, exploring how changing a variable in a turtle graphics program affects the output, or how changing the coefficients of a quadratic alters its graph and solutions, or even trying out possible answers for an online quiz; interactivity means pupils can investigate invariant properties of geometric constructions in dynamic geometry software; automatic functions means pupils can get immediate feedback on their answers to quiz questions.

Loveless's list might now be extended to include adaptation, as rapid advances in machine learning have resulted in systems that can quickly change their operation in response to the data provided; thus, for example, the questions pupils are asked in a quiz or game can be automatically changed to provide additional practice or further challenge according to that individual's past performance or the performance of other pupils on similar exercises, mirroring the way in which a sat nav might change its directions according to current position or road conditions, or a media recommendation system can base its advice on what other, similar users have liked.

Given how well suited digital technology is for at least some aspects of mathematics education, it is unsurprising that empirical data shows measurable impact on test scores:

> "The evidence for learning in mathematics is broadly positive, with effect sizes suggesting a small to medium impact. These findings compare to Hattie's (2017) effect size for the use of technology in mathematics of 0.33. The reported effect sizes vary from 0.15 to 0.61 for primary education and from -0.1 (non-significant) to 0.65 for secondary education." (Lewin et al. 2019, p10)

Lewin et al. (2019) indicate in particular that "simulations, scaffolding and intelligent tutoring systems lead to the greatest impact on mathematics attainment" (p11), although in UK schools at present there is limited use of software designed specifically for these applications.

Beyond the simple metric of test scores, there are likely to be other benefits to the use of digital technology in mathematics education, such as supporting independent enquiry, exploration of ideas beyond the scope of the exam specification, cross-curricular activities and collaborative approaches, but again there seems little work done to focus on these areas at present.

There are perhaps three elements to mathematics education at school level: teaching mathematics, learning mathematics and doing mathematics. These are closely connected: pupils' learning of mathematics is a result of their teachers teaching it, and happens through their actually doing mathematics. The purpose of both teaching and learning in mathematics should, at least partly, be that pupils get better at doing mathematics. Digital technology can play a role in all three of these elements.

Teaching mathematics

The digital technologies used by teachers themselves in mathematics lessons mirror those used for other subjects. Thus mathematics teaching is done:

- using standard presentation software;

- by sharing a quiz with the whole class using a generic platform such as Kahoot;
- by modelling calculations or the use of manipulatives using interactive whiteboards or visualisers;
- by providing pupils with exercises, sometimes via worksheets produced digitally by the teacher or through electronic versions of textbooks, but also through online forms;
- using online platforms or virtual learning environments (e.g. Google Classroom or Microsoft Teams), which typically include tools for online assessment;
- via video conferencing software (e.g. Zoom or Meet) or bespoke tutoring software; and
- through multimedia including narrated presentations, video recordings of worked examples, or educational videos.

Mathematics teachers might also use digital technology specifically designed for mathematics teaching, for example interactive visualisations, virtual manipulatives, or by modelling how pupils might use some of the digital tools for *doing* mathematics (see below). Teachers make use of dynamic geometry software on interactive whiteboards and display screens to model geometric construction, the properties of shapes, symmetry, and transformations.

Digital technology can also support inclusion in the mathematics classroom, e.g. through the use of automatic captions during lessons, or the automatic translation of lesson material into pupils' first language.

Learning mathematics

Pupils' own use of digital technology to learn mathematics often mirrors the way their teachers use it, so pupils will engage with online learning environments, respond to quizzes during lessons or for homework, participate in video lessons or tutorials, and access other online content as directed by their teacher.

Whilst much of pupils' work remains confined to paper, pupils may answer questions on screen, typically receiving automatically generated feedback immediately. Practice questions can be chosen from a bank of prepared materials or generated randomly using a pre-determined pattern, allowing for unlimited practice. Adaptive learning approaches, and more recently the application of machine learning, can create tailored practice material, taking account of a pupil's prior attainment and current targets. School mathematics seems particularly well suited to this sort of approach - it is straightforward for computers to generate questions automatically, answers are right or wrong, and more sophisticated approaches allow some ways for the computer to check mathematical reasoning. Some software attempts to 'gamify' learning, offering multimedia content, scores or badges alongside the questions themselves. This sort of drill and practice is the *only* application of digital technology noted by Ofsted in their recent review of research in mathematics education:

> "Computing technology can also help pupils acquire number facts by providing them with enough repetitions and direct feedback in ways that they enjoy." (Ofsted, 2021)

It is helpful to distinguish between short, tightly focussed online quizzes that follow on from direct teaching and worked examples, which might be considered the digital equivalent of the worksheet, and more integrated, adaptable intelligent tutoring systems which provide a variety of practice questions blending spaced repetition of previous topics with carefully structured new content. The former fits in well with conventional approaches to mathematics education, the latter is suited to more individualised approaches, perhaps in the context of homework, or indeed home schooling, and might be particularly beneficial for pupils working far below or above the level of their class. Ofsted recommend the former approach:

> "Pupils also experience more progress and enjoyment of computer maths games when core content is introduced as separate learning components that are systematically followed by 'mini-games'" (Ofsted, 2021)

Beyond this narrow approach, digital technology could also allow pupils to pursue topics independently, through reading about mathematical topics of interest, interacting with virtual manipulatives, watching video tutorials, finding interesting problems and puzzles and reading or watching others' solutions to these, or connecting to other learners.

Again, digital technology can support inclusion, for example presenting exercises in different formats, including via text to speech and automatic translation, and allowing responses in other forms.

Doing mathematics

For pupils, the experience of *doing* mathematics at school is mediated through particular technologies and the technologies used form pupils' internal model of mathematics (Drijvers et al., 2009). For the majority of the time pupils do mathematics in school, the technology used is pencil (or pen) and paper, thus understandably, but wrongly, giving the impression that mathematics is concerned mainly with methods for performing manual calculation. Recent years have seen more focus on physical manipulatives as an embodied rather than abstract way of thinking about mathematics.

Calculators are no longer on the primary curriculum, or required for the 'SATs.' The use of scientific or graphing calculators comes during secondary education, with more explicit encouragement for its use at A Level (Button, 2020).

There seems a big gap here between the nature of mathematics as an academic domain and its wide practical application for understanding systems and solving problems, where advanced, automated computation is the rule rather than the exception, and most pupils' experience of mathematics at school level. At school, the focus all too often seems to be repeated practice in performing computations that can be reliably and efficiently carried out by computers.

Over the years, there have been many experiments with using digital technology as a medium for pupils' own mathematics. Perhaps most significant, and distinctive, is the work of Papert (1980) and others using turtle graphics and other 'microworlds' in the **Logo** programming language. Papert made bold claims for the potential of a constructionist approach to learning mathematics through programming, and his vision was instrumental in the subsequent development of **Scratch** (Resnick, 2017) and the inclusion of turtle graphics libraries in Python and other languages. However, the experience of Logo programming in the mathematics classroom rarely lived up to his vision, typically being confined to an investigation of the external angles of regular polygons (Agalianos et al., 2006).

There are some indications of a renewed interest in using programming as a medium for pupils' own mathematics. Now that pupils are learning to program as part of the computing curriculum, they can use their programming skills in mathematics. This is easier in primary schools, where typically the same teacher takes both mathematics and computing lessons, and there have been successful approaches here, e.g. **Scratch** Maths (Noss et al., 2020). In secondary schools, pupils learn a text-based language, usually **Python**. Some computing teachers encourage pupils to apply their programming skills to relatively sophisticated mathematics problems (e.g. Project Euler[1]), but there seems relatively little use of programming in maths lessons at present, perhaps reflecting the absence of such approaches in the national curriculum or exam specifications. A notable exception to this general pattern is MEI's (2021) data science unit, using Python, Pandas and Jupyter Notebooks on Kaggle for visualising and analysing A Level large data sets.

Spreadsheet software has potential applications as a tool in mathematics education, typically through the use of formulae to model the relationship between variables, but also for numerical methods and linear programming. Spreadsheet software is commonly used for data visualisation including with large data sets. Most A Level teachers appear to favour a spreadsheet-based approach to the large data set material for A Level mathematics.

Some use is made of interactive **data visualisation software**. The National Curriculum offers non-statutory guidance that pupils should use 'coordinate-plotting ICT tools' (DfE, 2013, p134), and pupils might use visualisation software for creating charts of survey results or large data sets. Many of these programs include some statistical analysis tools.

Pupils may make use of **virtual manipulatives**, including dynamic geometry sketches shared by their teachers. For a generation of children familiar with touch screen interfaces from an early age, such virtual manipulatives have many

[1] https://projecteuler.net/

of the same affordances as their physical equivalent, although virtual manipulatives can be used to explore a far broader range, and depth, of mathematics than their physical equivalents. Virtual and augmented reality offer even more possibilities, although current implementations for schools are expensive and inflexible.

Computer algebra systems, such as Mathematica and the associated Wolfram Language, offer a set of tools which encompass and go beyond many of the above ways in which pupils might use digital technology to do mathematics. These programs allow the user to do most of the arithmetic, algebra and calculus required at school level, as well as providing an interactive interface to explore what happens if parameters are changed. Teachers may fear that, as with calculators in primary schools, the software does too much of the mathematics, but advocates such as Wolfram (2020) argue that software such as this moves the focus to framing problems and evaluating solutions rather than the more mechanical, operational aspects of, for example, calculating, solving equations or integration.

Looking ahead

Technological developments are hard to predict, but the next five to ten years may see an expansion of the tools available for teaching and assessing mathematics in platforms such as Google Classroom and Microsoft Teams / OneNote; further application of machine learning to adaptive learning design and intelligent tutoring systems; more flexible approaches to VR or AR to support mathematics teaching and learning; and the use of AI systems to check, and offer support for, pupils' mathematical reasoning.

The role of computing in real word mathematics is likely to grow ever more important, and thus the gap between real- and school- mathematics will widen, until there is a change in the current focus on didactic approaches, in which digital technology is largely limited to presenting content, setting and marking questions, and performing numerical calculations. The inclusion of programming in the computing curriculum already makes it easier for teachers to use programming based approaches for pupils doing mathematics, allowing a return to exploratory, constructionist approaches, making effective use of computer algebra systems and using data science approaches to school statistics.

Whilst individual examples of these more mathematically authentic approaches at school level can be found, it seems unlikely that there will be widespread adoption of these approaches without some explicit encouragement from the Department for Education, Ofsted and / or the NCETM, changes in assessment at 11+, 16+ and 18+, or robust evidence that these approaches are *more* effective in supporting pupils' progress under existing assessment regimes than traditional approaches.

Suggested questions for further exploration

The key question in determining the role of digital technology in mathematical futures is whether we should use its potential to provide ever more efficient ways of teaching pupils to do computations by hand, or whether we should change the nature of school mathematics to reflect the way real mathematics has already changed due to the automation of those computations?

- Is a change needed, or are current, largely didactic approaches actually the most effective way to teach and learn mathematics?
- Which schools are already using digital technology well for *doing* mathematics? Could case studies be compiled of their successes?
- What approaches to the use of digital technology in mathematics education have worked well in other jurisdictions?
- What are the barriers to pupils making more use of their existing programming skills in mathematics lessons?
- What changes in assessment regimes would promote more authentic use of digital technology for mathematics?

References

Agalianos, A., Whitty, G. and Noss, R. (2006) The social shaping of Logo. *Social studies of science*, 36 (2): 241–267.

Button, T. (2020) *Ten calculations A Level students should do on a calculator*. Available at: https://www.ncetm.org.uk/features/ten-calculations-a-level-students-should-do-on-a-calculator/.

Department for Education (2013) *The national curriculum in England: Framework document*. London: Department for Education.

Drijvers, P., Kieran, C., Mariotti, M.-A., et al. (2009) "Integrating technology into mathematics education: Theoretical perspectives." In *Mathematics education and technology-rethinking the terrain*. Springer. pp. 89–132.

Lewin, C., Smith, A., Morris, S., et al. (2019) *Using digital technology to improve learning: Evidence review*. London: Education Endowment Foundation.

Loveless, A. (2002) *Literature review in creativity, new technologies and learning*. 4. Bristol: Futurelab; Futurelab.

MEI (2021) *Introduction to data science*. Available at: https://mei.org.uk/introduction-to-data-science/.

Noss, R., Hoyles, C., Saunders, P., et al. (2020) "Making constructionism work at scale: The story of Scratchmaths." In Holbert, N., Berland, M. and Kafai, Y. (eds.) *Designing constructionist futures: The art, theory and practice of learning designs*. Cambrdige, MA: MIT Press. pp. 39–52.

Ofsted (2021) *Research review series: mathematics.* London: Ofsted. Available at: https://www.gov.uk/government/publications/research-review-series-mathematics/research-review-series-mathematics.

Papert, S. (1980) *Mindstorms: Children, computers and powerful ideas.* New York: Basic Books.

Resnick, M. (2017) The seeds that Seymour sowed. *International Journal of Child-Computer.*

Wolfram, C. (2020) *The math(s) fix: An education blueprint for the AI age.* Wolfram Media, Inc.